Deformation using Agents for Map Generalization - Application to the Preservation of Relationships between Fields and Objects

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Abstract

Some research in map generalization techniques using the agent paradigm have allowed to progress significantly toward automation. These models allow the computation of discrete transformations of the objects but are not adapted for continuous transformations such as deformations. Our challenge is to allow these models to manage such continuous transformations.

As an application, we aim to introduce field data such as the relief in agent-based generalization models. Many relationships exist between fields and objects (especially buildings, roads and rivers). Our concern is to preserve these relationships. We propose to allow the geographic agents to deform a field during the generalization process.

To attain this objective, we propose a model:

- To measure and interpret relationships between fields and objects (this allows to define new constraints),
- To compute fields deformations under objects stretching,
- To manage the automatic trigger of these measurements and deformations in the existing agent based generalization models.

We propose to model the field as a triangulation, and to explicit some shape preservation constraints on the points, segments, angles and triangles composing the field. When a point is stretched, its displacement propagates to its neighbors using a multi-agent convergence method we have built.

I. Introduction and context

Cartographic generalization is the process of transformation of map objects to make them legible for a specific scale. Operations such as dilation, displacement, elimination, etc, are applied on the objects, to make them satify their legibility constraints. Generalization is an information synthesis process: some important details must be preserved or emphasized, while other must be erased. Map generalization automation attemps to produce a map for a target scale from vectorial geographic databases. The issue of the automation is to improve map production from geographic databases: updating propagation from geographic databases to map would be faster, and on-demand maps for specific customers needs could be easier produced.

Works in generalization automation mainly focus on the conception of algorithms for geometric transformations, measures for caracterisation, generalization models (triggering processes) and assessment of generalization. Our work concerns generalization models. Several approaches have been studied.

Optimization based models adopt an holistic approach. A balance position between all the constraints of the map is found using a global resolution method (finite elements method or least square adjustement). This methods allows to compute a continuous transformation of all the objects of the map to make them satisfy their constraints. As applications of this approach, we could mention the use of flexible triangles modelisation (Hojholt 2000), least square adjustement (Harrie and Sarjakoski 2002; Sester 2005), snakes (Burghardt and Meier 1997; Galanda 2003; Guilbert et al. 2006) or elastic beams (Bader 2001).

Other models use a discrete transformations approach. The principle of this models is to proceed step by step to solve specific cartographic conficts: a generalization engine detects the conflict to solve, chooses the right algorithm and triggers it. The principles of this approach are presented in (McMaster and Shea 1988; Shea and McMaster 1989; Brassel and Weibel 1988). Some agent based generalization models such as the AGENT (Ruas 1999; Barrault et al. 2001) and the CartACom model (Duchêne 2004 a) follow this approach. Our work is based on these models. An agent is defined as *"a computer system that is situated in some environment, and that is capable of autonomous action in this environment in order to meet its design objectives"* (Weiss 1999, 29). The principle of these models is to consider the map objects (such as buildings, roads…) as agents: the purpose of each agent is to satisfy its constraints (following the approach of (Beard 1991)). To reach this purpose, each agent is able to analyse himself its own state: it measures its characteristics and checks if the level of satisfaction of its constraints is good. Then, depending on its unsatisfied constraints, the agent chooses a suitable generalization algorithm to apply to himself, in order to progress toward the global satisfaction of its set of constraints. During its generalisation process, the agent controls the evolution of its state and is able to backtrack to a previous state if a treatment failed to improve its state. These models consider several levels of objects: the so-called "micro" and "meso" levels. The "micro level" is the level of the individual objects (such as the buildings, the roads…). The "meso level" is the level of groups of objects (building alignments, buildings of a urban block…). Constraints can be carried by micro objects (for example, the size of a building) and by meso objects (for example, the alignment of a building set). In this paper, we call "micro objects" the object considered individually, such as a building, a road, according to (Ruas 1999).

Most of the generalization models proposed have been applied mainly to the generalisation of roads and buildings. We propose in this paper to take into account fields in the agent-based generalization models previously presented. A field could be defined as a phenomenon which allows assigning a value to every location of the geographic space (Cova and Goodchild 2002). In this paper, we consider the relief and the land use cover as fields. A specificity of these objects in the generalization process is that they should be subjected to continuous transformations, rather than discrete. Both transformations are useful in map generalization. The limit of agent-based models is their non-adaptation to compute such continuous transformations.

This paper presents a way to compute such continuous transformations to the fields in an agent context. We present an application to the preservation of relations between fields and objects. We propose to constrain the generalization process to allow a preservation of these relationships by deforming the fields.

The part II presents an example of object-field relationship preservation constraint. We show that a deformation of the field is necessary. Then, we present our general approach to manage field objects, and then we show the limits of the agent-based models to manage deformations.

The part III presents our proposition to compute continuous transformations of fields using the so-called "sub-field objects". We give a way to trigger such continuous transformations from the existing agent-based generalization models.

II. The issue

II.a. An example case

We introduce the issue with a simple example: a building and a road (Figure 1a). When generalizing this situation, the building dilates itself to satisfy its size constraint (Figure 1b) and must therefore be displaced to satisy its proximity relation with the road (Figure 1c). Because of this displacement, its altitude seems to have changed. We propose to constrain the preservation of the altitude of this building.

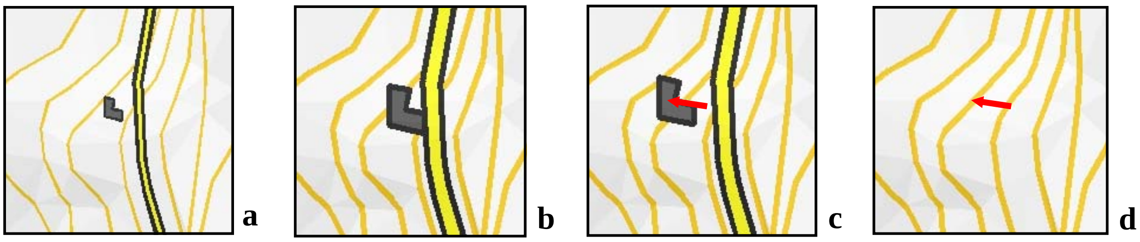


Figure 1. *An example case: the altitude of the building changes when it generalizes.*

This constraint concerns the relationship between an object (the building) and a field (the relief). The altitude of the building is not a static attribute: it is given by its localisation on the relief.

How to preserve this relationship? The building can't be changed because of its own constraints (size and proximity to the road). An other solution would be to act on the field.

How to change the field? The relationship could be preserved by stretching a point of the field to get the altitude of the building as near of its initial value as possible (Figure 1d). The field has some shape characteristics to preserve (for example, structure lines of the relief). This displacement must be diffused to respect the relief's own shape preservation constraints.

This situation is overconstrained: it is not possible to preserve totally both the shape of the relief and the relationship with the building. A balance between the inner shape constraints of the field and the object-field constraint must be found. The deformation can be seen as a result of this balance position research.

We present now a proposition of a general framework allowing to consider object-field relationships preservation contraints in the generalization process.

II.b. Object-field relationship preservation constraints

The previous part gave an exemple of object-field relationship. We could generalize this case to other object-field relationships.

A lot of relationships exist between the micro objects (buildings, roads) and the fields (relief, land use) as presented in (Gaffuri 2005): the repartition of micro objects on the map is constrained by the fields. For example, the road sections in a mountainous area are built to have a specific slope, road sections are often limits of land use parcels, buildings have a specific altitude, and are located in a specific type of land use parcel… These relationships between micro objects and fields can be broken when generalizing the micro objects.

Figure 2 gives our proposition to take into account the field objects in a generalization process. We propose:

- to differentiate, among the map objects, the micro objects (roads, buildings...) and the fields (relief, land use cover...),
- to characterize the relationships between the objects and fields,
- to allow the objects to deform the fields and the fields to constrain the objects, in order to preserve the relationships between fields and objects.

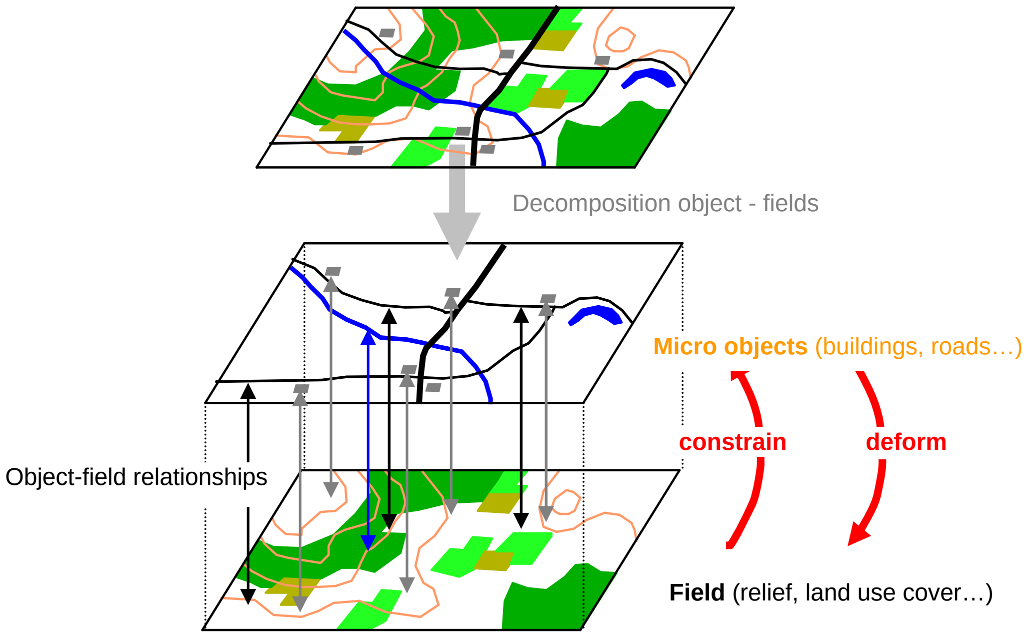


Figure 2. *Our framework proposition for the preservation of object-field relationships*

The deformation of the field schould not be a simple isotropic diffusion caused by the displacement of a point of the field. It should take into account shape specifities of the field. From the field point of view, the relationship constraint appears like an external constraint that stretches one of its points, and the shape preservation constraints are inner preservation constraints. The deformation of the field is the result of the research of a balance position between the external and the inner constraints of the field (Figure 3).

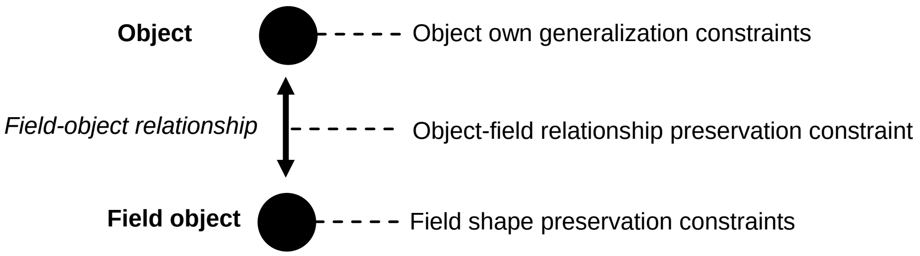


Figure 3. *The constraints to consider*

To make these constraints preservation possible, we have to allow the management of continuous and discrete transformations jointly, in a the same process. We present the problem in detail in the following part.

II.c. Discrete and continuous changes in map generalization

During the generalization process, two kinds of transformation are considered.

- Continuous transformations: these transformation correspond to a smooth change of the objects. The objects are deformed to satisfy progressively their constraints.
- Discrete transformations: these transformations correspond to a violent change of the objects. Operations such as elimination, big displacements, aggregation, collapse, typification... are discrete transformations.

The kind of transformation useful for a given generalization process mainly depends on two factors:

- The level of detail change: when the level of detail change is little, continuous operations can be sufficient to satisfy the cartographic constraints. The bigger the level of detail change is, the more discrete transformations are needed, as illustrated by (Van Kreveld 2001).

- The nature of the objects: depending on their nature, geographic objects do not behave the same way during the generalization process. Some of them are much more deformed (fields, networks...) while other are rather curtly treated (buildings, clusters...). (Harrie & Sarjakoski 2002) make a distinction between “rigid” and “plastic” objects. Continuous transformations are much more adapted to plastic objects (fields) while discrete transformations are suitable for rigid objects (buildings).

It seems impossible to determine a priori wich kind of transformations will be needed for a generalization process. Often, both discrete and continuous transformations are needed for some objects. For example, a road section can be subjected to continuous transformations such as deformation, and to discrete transformations, such as the bend removal operation. For a field object such as the relief, transformations could be continuous (for example, the smoothing algorithm presented in (Gold & Thibault 2001)), or discrete (like the thalweg elimination algorithm of (Ai 2004)). These examples show that a complete generalization system should be able to manage both types of tranformation.

As presented in the introduction, optimization techniques allow continuous transformations while agent-based allow discrete ones. The reason of this is inherent to the way these models have to satisfy a set of constraints. In optimization based models, a balance position between constraints is searched; constraints are considered as “elastic constraints”. In agent-based models, the constraints are satisfied step by step. Some of them are considered as much more important than others and are therefore completely satisfied while the others are relaxed: there is no balance position. As seen previously, a deformation is the result of a balance position between external stretching constraints and inner shape preservation constraints. So we schould use a method to get a balance between constraints, such as the ones used in optimization (finite elements methods and least-square adjustement). The problem is that these resolution method are too closed and can not manage discrete transformation.

To give to the agent-based generalization models the capability to manage continuous transformations, we aim to give them the ability to find a balance between elastic constraints. We propose an enrichment of agent based models for continuous deformations using the approach of (Kocmoud and House 1998) for cartogram construction. We present now our model and an application for the field deformation.

III. Proposition of deformation model

III.a. Principles

The principles of our model are the three following points:

- The field is decomposed into parts (points, segments, angles and triangles) carrying elastic shape preservation constraints (Figure 4). The association of these inner constraints of the field allows to constrain the general shape of the field. Because they are parts of the field, we propose to call these objects “sub-field objects”. The data structure of this decompostion is given Figure 5.
- The micro objects have the ability to stretch points of the field to satisfy their field relationship preservation constraint. This stretching of the field is done through an elastic constraint between the micro and points of the field. We propose to call this external constraint of the field “stretching constraint”.
- Points are modelled as agents: to compute a deformation by finding a balance position between the inner and external constraints of the field, we propose to consider the points of the field as agents, that are autonomous to compute their own displacement.

The deformations are triggered after the generalisation of each micro object. The micro objects act on the field by stretching points. When a point is stretched, it is not in a balance position anymore. Our method allows to displace this point and its neighbours until a balance position between the stretching and the shape constraints is attained. The displacement of the point is diffused; the field is deformed.

The example shown in this article concerns the relief. For the triangulation of this field, we have chosen to merge the contour lines in the triangulation (Figure 4). This could allow to constrain the shape of the contours, as explained later.

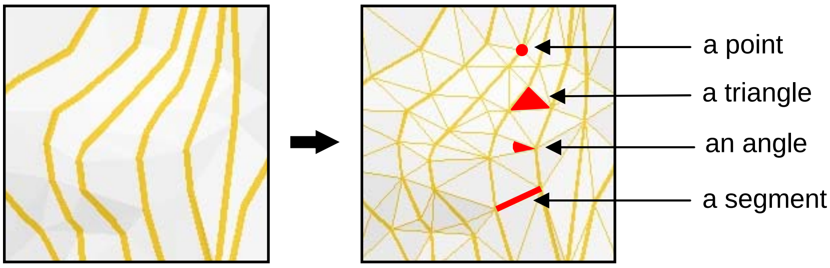


Figure 4. *Decomposition of the field into sub-field objects.*

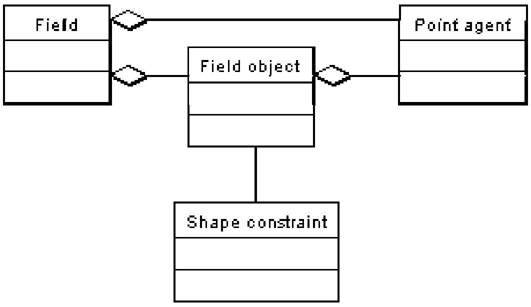


Figure 5. *UML class diagram of the field decomposition objects.*

We detail now how an agent point belonging to a field reaches its balance position, and then how the micro objects manage the stretching of the field.

III.b. Points as agents

To find a balance position of the points, we propose to consider the points of the fields as agents: each point is "alive". It is able to analyse itself and moves in order to reach a balance position between its constraints. Its constraints are all the constraints of the sub-field objects it belongs to. Each constraint forces the point to do a displacement to progress toward its satisfaction (as an elastic constraint). We give some details about the way to calculate these displacements in III.c.

As an agent, a point has the following methods:

checkBalance(): For each constraint of the point, the point computes the displacements needed so that this constraint is integrally satisfied. If the length of the displacement is less than a small threshold (equal to the resolution of the data for example), it is considered as null. Then the point is in a balance position. Figure 7 page 8 gives a representation of a point in its balance position.

computeDisplacement(): the point computes a displacement for each constraint. This is not the displacement to do in order to integrally satisfy the constraint, but only a certain part of this displacement, in order to progress toward the satisfaction of this constraint. The sum of these displacements allows to progress toward the balance position of the point.

progressTowardBalance(): displace the point toward the displacement given by the method computeDisplacement().

To find its balance position, the point progresses until it reaches its balance position. All the points try to progress toward their balance position together.

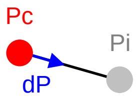
We give now further details on the computation of the displacement of each shape preservation constraint.

III.c. Shape preservation constraints

Our model does not use a “force” model to determine the displacement caused by each constraint of the point: the displacement is directly computed. For each constraint, the point is able to determine the displacement to do in order to completely satisfy the constraint. Because the point has several constraints and aims to find a balance position between them, the displacement is only a part to progress toward the constraint satisfaction. This part depends on the number of constraints the point has. If a point has N constraints, the displacement of each constraint will be $1/N$ of the displacement needed to completely satisfy the constraint. Now we detail the shape preservation constraints used, and how the displacement caused by them are computed. (Some of them are an adaptation of the springs used in (Kocmoud and House 1998)).

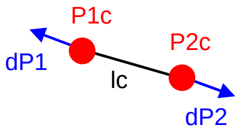
Notation: Pi is a point in its initial position, Pc in its current position, $dP(dx,dy)$ is the part of the displacement to do to progress toward the constraint satisfaction.

Initial position point constraint : (aims to preserve the initial position of a point)



$dx = 1/N * (Pi.x - P.x)$
$dy = 1/N * (Pi.y - P.y)$

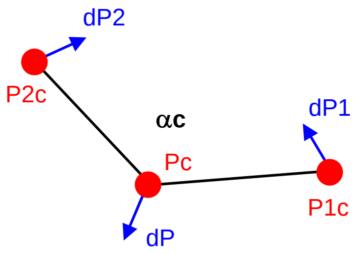
Segment length constraint: (aims to preserve the initial length of a segment)



$dx1 = 1/2N * (li/lc - 1) * (P1c.x - P2c.x)$
$dy1 = 1/2N * (li/lc - 1) * (P1c.y - P2c.y)$
$dP2 = -dP1$

li=initial length of the segment; lc=current length of the segment.

Angle value constraint: (aims to preserve the initial value of an angle)



$dx1 = p.x - p1.x + \cos(d\alpha) * (p1.x - p.x) - \sin(d\alpha) * (p1.y - p.y)$
$dy1 = p.y - p1.y + \sin(d\alpha) * (p1.x - p.x) + \cos(d\alpha) * (p1.y - p.y)$

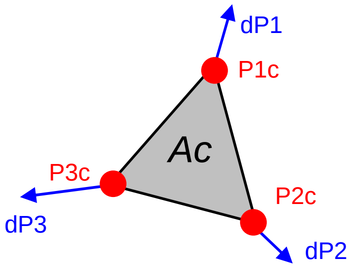
$dx2 = p.x - p2.x + \cos(d\alpha) * (p2.x - p.x) + \sin(d\alpha) * (p2.y - p.y)$
$dy2 = p.y - p2.y - \sin(d\alpha) * (p2.x - p.x) + \cos(d\alpha) * (p2.y - p.y)$

$dx = -dx1 - dx2 \quad ; \quad dy = -dy1 - dy2$

With $d\alpha = 1/2N * (\alpha c - \alpha i)$

αi= initial value of the angle; αc= current value of the angle.

Triangle area constraint: (aims to preserve the initial area of a triangle)



$$dx1 = 2/3N * (Ai - Ac)/P2cP1c^2 * (P2c.x - P1c.x)$$
$$dx1 = 2/3N * (Ai - Ac)/P2cP1c^2 * (P2c.y - P1c.y)$$

Analog calculus for dP2 and dP3.

Ai= initial area of the triangle.

Ac= current area of the triangle.

The displacement of each point influences the balance of its neighbours. Then, this displacement propagates until each point has reached its balance position. The result is a deformation of the field.

We present an example of such a deformation Figure 6. We simulate a displacement of a point (the red arrow figure a). This displacement is progressively diffused to its neighbours (figures b and c). The result is a deformed field (figure d), where each point has reached its balance position.

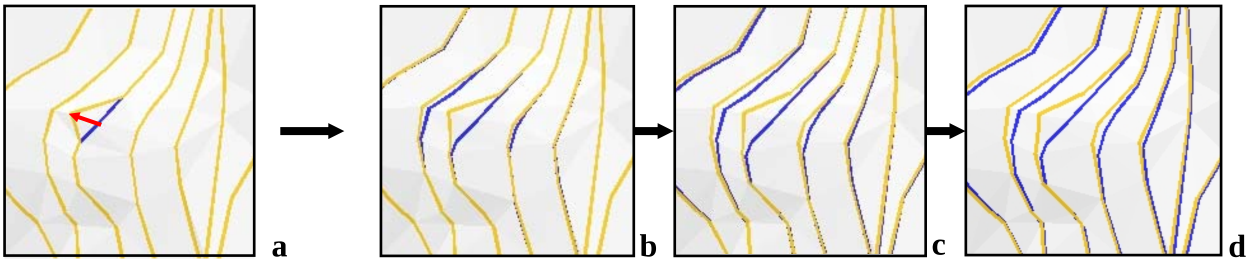


Figure 6. *Progressive diffusion of a point displacement (a) to its neighbours by constraint balance of the sub-field objects (b)(c)(d). (in orange: the contour lines; in blue: the contour lines in their initial state)*

Figure 7 represent a zoom on a point in its balance position at the end of the process. Each purple line represents a displacement proposed by each constraint of the point. The sum of these displacements is null.

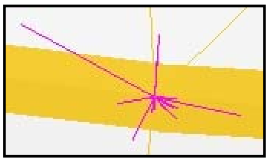


Figure 7. *A point in its balance position, and its displacements*

For a point having N constraints, the obtained position is a balance position between all the constraints of the point. It is possible to define a relative importance of the constraints by tuning a weight value for each constraint in order to obtain a weighted balance position. In the presented case, the weights of the constraints are all equal (the value of the weight is 1/N). It is possible to define an importance value for each constraint. The weight of a constraint is then determined from its importance value and the importance values of the other constraints of the point by the following way:

$$\textit{weight} = \textit{importance} \,/\, \textit{sum of the importances}$$

When the importance values of the constraints are all equals, the value of the weight is 1/N.

With this way, it is possible to insist on some specific shape characteristics preservations. For example, we could choose to increase the importance of the contour line shape preservation constraints, because these shapes are cartographically important.

Eventually, concerning the trigger of the point agents, it is not necessary to activate all the agents of the field to compute such a deformation: when a point is stretched, only some of its neighbours will move to diffuse the stretching effect. For this reason, each point is in a passive state and becomes active as soon as it is "waken" by a stretching from a micro object, or by one of its activated neighbour points. This way to activate the point agents allows to make local deformations during the generalization process. The deformation is not obtained with huge time-consuming global method on the whole field.

We present now how a micro object can stretch the field.

III.d. Stretching constraint

As shown previously Figure 1, when a micro object changes during the generalisation process, a relationship it has with the field can be broken. To constrain this relationship, we need to measure it, and tune how we want this relationship to be constrained. For that purpose, we need an interpretation of the difference between the current and the initial state of the relationship, following the approach of (Ruas 1999). Figure 8 shows an example for the building altitude preservation constraint presented previously. The altitude of the building is measured on the DTM representing the relief. The violation of this constraint is given by the value *Δalt* and interpreted with the satisfaction graph.

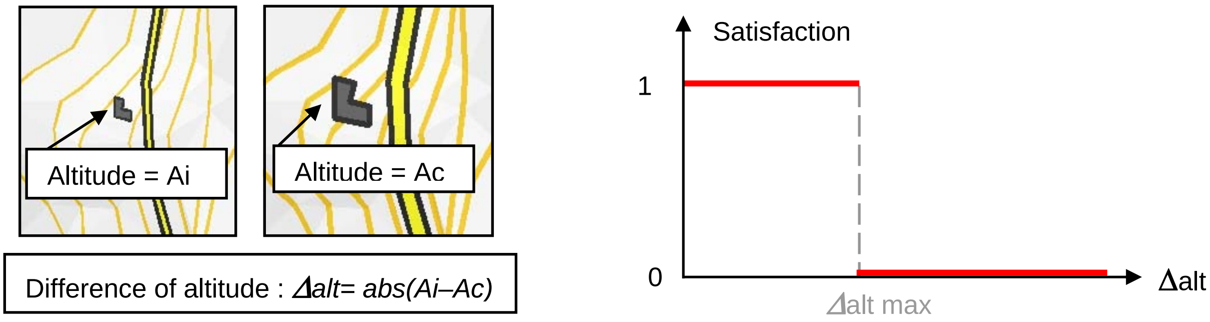


Figure 8. *Satisfaction of the altitude constraint depending on the variation of altitude.*

The satisfaction graph allows tuning how we want the relation to be constrained, depending on the map specifications. The satisfaction is given by a value between 1 (satisfied) and 0 (unsatisfied). For our example, we propose a stair-like graph (Figure 8, on the right). Under the value *Δalt max*, the altitude change is considered as not significant: the satisfaction is 1. Then, the constraint is considered as unsatisfied. For example, the Figure 9a shows the building whose altitude is 824m. We suppose we want constraint the building to keep an altitude precision of 3m. On the Figure 9b the building has generalized itself. Its altitude is now 818m: the satisfaction value is 0; the constraint is considered as violated.

To preserve the altitude constraint, the building has the capability to measure the satisfaction of its altitude preservation constraint after its own generalization, and then to stretch the relief field to make its constraint satisfied again. We propose to compute this stretching of the field from the initial position of the center of the building (Figure 9a) in the field. To stretch this point, we propose to use a segment with a length preservation constraint (as presented in IIIc). This segment links a point of the DTM and the center of the generalized building, and allows the building to stretch the field. Because it is not possible to determine a priori which weight value to choose for this stretching constraint, the building tries several values until the constraint is satisfied again. The result is shown Figure 9c. The relief has been deformed by the building (the initial contours are drawn in blue) in order to correct the building's altitude (according to the satisfaction graph).

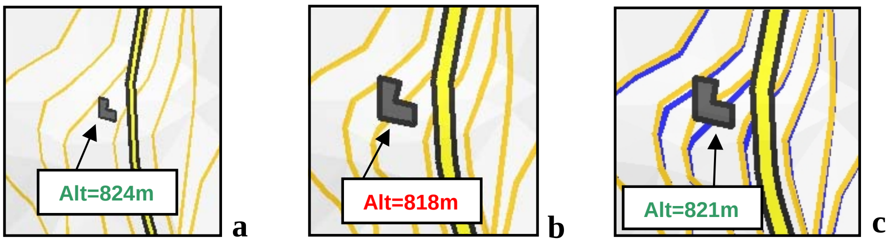


Figure 9. *The building altitude preservation constraint*

In this process, the field does not constrain the micro object. A further objective would be to allow the field to control its state after the stretching of the object, in order to avoid some possible too important field deformations. If the field considers it is too much deformed, it should refuse this deformation and constrain the micro objects to change.

IV. Conclusion and further work

In this article we have proposed an agent-based model to compute continuous deformations. This model allows to use discrete and continuous transformations jointly, as showed for the object-field relationship preservation. This work is an example of a way to manage discrete and continuous transformations in the same generalization process. Because we need both transformations in the generalization process, the association between the generalization models seems as important as the models themselves. Agent-based modelling allows to obtain open resolution methods, which is useful for this purpose. Further work could be done to progress toward a merging between continuous and discrete transformation models as we propose in (gaffuri 2006).

We could propose to apply our method to other deformable objects. For example, for the generalization of networks, we could define specific road section and crossroad shape preservation constraints to improve the diffusion of discrete transformations. The need to compute both continuous and discrete transformation for networks generalization is high.

Concerning the micro object-field relationship conservation, further work could be developed. We gave one proposition for the building altitude preservation. In our example, the building is considered as a point. Some other constraints between other objects, especially network sections, would be much trickier, because these objects are deformable too, and should deform the field through several points. Some works proposed a way to merge roads section in a DTM for map generalization (Kremeike 2004). Some characterisation measures on the profile of the section as proposed in (Plazanet and Spagnuolo 1998) could be used.

We also observed that some constraints between fields and objects are sometimes hard. For example, the topologic relationship between a road section and a land use parcel, or the position of a river flowing in a thalweg are important relationships. The stretching constraints for the relationships preservation constraints concerning these cases should not be elastic.

Finally, we could improve the proposed field deformation process by adding other sub-field objects and other shape preservation constraints. Another method could be proposed to determine which point of the field to stretch to deform it for a specific purpose. As we proposed, we could use a characterisation method to determine when a field is too much deformed. In this case, the field could constrain the stretching object to change. For the choice of relative importance of the shape preservation constraints, we choose the same value for each constraint. We could imagine allowing the field to tune itself these importance values depending on its state: if the field measures a too high deformation of some of its sub-field objects, it could constrain some specific shape preservation constraints to be much more important.

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